Dielectric Constant and Loss Tangent of Microwave Ferrites at Elevated Temperatures*

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Summary—**Experimental data are given on the effect of elevated temperature on the dielectric properties of ferrites intended for microwave applications. Measurements were made at X band. The real part of the dielectric constant generally increases slowly with temperature, the maximum temperature coefficient observed being 300 parts per million per degree C. The dielectric loss tangent also generally increases with temperature. Measurements were made in a rectangular waveguide cavity, using a modified perturbation technique. It is shown that the simple perturbation technique may cause an appreciable error in the measurement of the real part of the dielectric constant. The effect of the finite resistivity of the cavity walls and the effect of the cavity irises on the measurement of the dielectric loss tangent, are also considered.**

Introduction

The DIELECTRIC properties of a ferrite strongly influence the performance of the microwave device in which it is used. It is therefore important to know how these properties vary with temperature. The dielectric properties of a large number of ferrites were measured over the temperature range of room temperature to 250°C or the Curie temperature, whichever was lower. The samples were obtained from several commercial suppliers, and were commercially available at the time they were obtained.

The dielectric constant of ferrites is a complex quantity given by

$$K = K' - jK'', \tag{1}$$

where K' is the real part of the dielectric constant, and K'' the imaginary or dissipative part. The dielectric loss tangent is defined as

$$\tan \delta = K''/K'. \tag{2}$$

The dielectric loss tangent is sometimes called the dissipation factor.

General Test Method

Measurements were made using a resonant cavity and perturbation theory[1–3] substantially as described in the "Proposed Method of Test for Complex Dielectric Constant of Non-Metallic Magnetic Materials at Microwave Frequencies," prepared by a task group of the American Society for Testing Materials (ASTM).

The cavity used was a TE_{10n} cavity in rectangular waveguide where n was odd. The test samples were in the shape of rods approximately 0.035 in in diameter and $1\frac{1}{4}$ in long. Holes with diameters slightly larger than 0.035 in were drilled in the center of the top and bottom walls of the cavity so that a sample could be inserted into the cavity. Both ends of the sample extended through the cavity. The sample was thus located at an electric field maximum, and the axis of the sample was paralled to the electric field.

The test frequency was approximately 9.4 gigacycles. All frequency measurements were made using a frequency counter that measured a known subharmonic of the actual frequency.

Test Results

Test results for the real part of the dielectric constant K' are shown in Table I. At 25°C the actual value of K' is given. At other temperatures, the ratio of K' at that temperature to K' at 25°C is given.

Table I shows that the real part of the dielectric constant generally increased with temperature. However, the increase was quite small. Sample 1 showed the maximum increase, and at 250°C this amounted to 6.2 per cent; this corresponds to a temperature coefficient of approximately 300 parts per million per degree C.

Test results for tan δ are shown in Table II. For most samples, tan δ increased with increasing temperature. Some cases of tan δ decreasing with temperature were noted. These decreases are fairly small and may be due to measurement error.

The composition of the samples and the values of saturation magnetization ($4\pi M_s$) were obtained from the manufacturer's data. The composition gives only the principal elements and may not include small amounts of additives. It will be noted that in some cases there are two samples, each with the same indicated composition and very similar saturation magnetization, that behave quite differently. This could be due to differences in processing techniques and use of additives as practiced by different manufacturers.

Discussion of Some Aspects of Test Method

Formulas Based on Perturbation Theory

In measuring the dielectric constant of a ferrite sample, the resonant frequency and loss tangent of the

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[1] G. Birnbaum and J. Franeau, "Measurement of the dielectric constant and loss of solids and liquids by a cavity perurbation technique," *J. Appl. Phys.*, vol. 20, pp. 817–818; August, 1949.

[2] J. O. Artman and P. E. Tannenwald, "Microwave Susceptibility Measurements in Ferrites," M.I.T. Lincoln Lab., Lexington, Mass., Tech. Rept. No. 70; October 27, 1954.

[3] R. F. Soohoo, "Theory and Application of Ferrites," Prentice-Hall, Inc., Englewood Cliffs, N. J., p. 103; 1960.

TABLE I

K' AS A FUNCTION OF TEMPERATURE

Sample	Composition	$4\pi M_s$*	25°C	100°C	150°C	200°C	250°C
			K'	Ratio	Ratio	Ratio	Ratio
1	Ni	3300	12.5	1.013	1.027	1.036	1.062
2	NiCo	3150	12 4	1.008	1.019	1.021	1.031
3	NiCo	3000	12.5	1 004	1.003	1.021	1.027
4	NiCo	2400	12.4	.998	1.012	1.024	1.032
5	Ni	2390	8.72	.994	1.009	1.016	1.023
6	MgMn	2300	12.3	1.009	1.018	1.016	—
7	MgMn	2300	12.7	1.007	1.017	1.013	1.034
8	MgMn	2000	12.2	1.006	1.019	1.021	1.037
9	MgMn	1900	12.5	1.007	1.019	1.022	1.032
10	YIG	1880	14.7	1.009	1.015	1.034	1.030
11	MgMn	1800	11.9	1.009	1.015	1.021	1.040
12	MgMn	1800	11.8	1.010	1.023	1.020	1.051
13	NiCoAl	1670	10.9	1.003	1.016	1.019	1.031
14	Ni	1500	8.46	1.005	.986	.981	1.029
15	MgMnAl	1500	11.7	1.000	1.014	1 026	—
16	NiCoAl	1400	12.0	.998	1.009	1.025	1.027
17	NiAl	1300	11.0	1.002	1.017	1.031	1.051
18	MgMnAl	1250	11.2	.994	1.013	—	—
19	MgMnAl	1200	11.3	1.000	1.000	1.025	—
20	MgMnAl	1200	10.7	.988	1.011	—	—
21	MgMnAl	1100	10.4	1.000	1.018	—	—
22	MgMnAl	1030	11.2	.989	1.012	—	—
23	MgMnAl	950	9.26	1.002	1 010	—	—
24	MgMnAl	950	11.2	1.002	1.014	—	—
25	MgMnAl	800	10.8	1.000	1.016	—	—
26	NiAl	750	10.1	.999	1 014	1.030	1.049
27	MgMnAl	700	9.60	1.006	—	—	—
28	Hyb. Garnet	670	14.1	1.005	1.003	—	—
29	MgMnAl	600	11.2	1.006	—	—	—
30	MgMnAl	500	10.8	.997	—	—	—
31	NiAl	440	8.06	1.000	1.020	—	—
32	NiAl	350	8.20	1.007	1.006	—	—

* Commercial Values

$$\text{Ratio}=\frac{K' \text{ at given temperature}}{K' \text{ at } 25°\text{C}}$$

TABLE II

TAN δ AS A FUNCTION OF TEMPERATURE

Sample	Composition	$4\pi M_s$*	25°C	100°C	150°C	200°C	250°C
1	Ni	3300	0137	.0205	.0318	.0447	.0715
2	NiCo	3150	.0011	.0006	.0008	.0007	.0009
3	NiCo	3000	.0013	.0008	.0006	.0007	.0075
4	NiCo	2400	.0001	.0004	.0006	.0012	.0022
5	Ni	2390	.0009	.0006	.0005	.0009	.0020
6	MgMn	2300	.0002	.0001	.0003	.0006	—
7	MgMn	2300	.0001	.0001	.0004	.0012	.0020
8	MgMn	2000	.0004	.0002	.0007	.0011	.0032
9	MgMn	1900	.0001	.0001	.0002	.0002	.0011
10	YIG	1880	.0007	.0006	.0006	.0006	.0006
11	MgMn	1800	.0004	.0013	.0025	.0034	.0096
12	MgMn	1800	0029	.0076	.0149	.0194	.0393
13	NiCoAl	1670	.0007	.0004	.0002	.0001	.0001
14	Ni	1500	.0014	.0018	.0011	.0017	.0021
15	MgMnAl	1500	.0001	.0003	.0007	.0017	—
16	NiCoAl	1400	.0005	.0005	.0006	.0007	.0011
17	NiAl	1300	.0043	.0071	.0106	.0165	.0244
18	MgMnAl	1250	.0001	0002	0005	—	—
19	MgMnAl	1200	.0009	.0003	.0006	.0011	—
20	MgMnAl	1200	.0008	.0016	.0027	—	—
21	MgMnAl	1100	.0009	.0008	.0013	—	—
22	MgMnAl	1030	.0001	.0002	.0005	—	—
23	MgMnAl	950	.0002	.0003	.0005	—	—
24	MgMnAl	950	.0002	0001	.0002	—	—
25	MgMnAl	800	.0001	.0002	.0004	—	—
26	NiAl	750	.0002	.0003	.0008	.0006	.0002
27	MgMnAl	700	.0004	.0003	—	—	—
28	Hyb. Garnet	670	.0014	.0011	.0009	—	—
29	MgMnAl	600	.0001	.0001	—	—	—
30	MgMnAl	500	.0003	.0005	—	—	—
31	NiAl	440	.0004	.0007	.0013	—	—
32	NiAl	350	.0003	.0007	.0004	—	—

* Commercial Values.

cavity without a sample, and of the cavity with the sample, are determined. Let subscript 1 denote measurements made on the cavity without a sample, and subscript 2 denote measurements on the cavity with a sample. We then have[1–3] (for the cavity described above),

$$\frac{F_1 - F_2}{F_1} = 2(K' - 1)\frac{V_s}{V_c} \tag{3}$$

$$(\tan \delta_c)_2 - (\tan \delta_c)_1 = 4K''\frac{V_s}{V_c}, \tag{4}$$

where F is the resonant frequency of the cavity, V_s and V_c are the volume of the sample and cavity, respectively, and $\tan \delta_c$ is the loss tangent of the cavity. K' and K'' can be readily determined from the above.

The loss tangent of a cavity is measured by determining the two frequencies F_h and F_l on either side of the resonant frequency F_r, at which the output of the cavity drops by α db. We then have

$$\tan \delta_c = \frac{F_h - F_l}{F_r\sqrt{10^{\alpha/10} - 1}}\,. \tag{5}$$

Eq. (5) is given in the ASTM test method referred to previously.

Correction for Relatively Large Sample Diameter

For the perturbation theory to be accurate the sample diameter must be "sufficiently small." The criteria as to when a diameter is sufficiently small will be discussed below. In practice, sample diameters in the range of 0.03 to 0.05 inch have been used by different laboratories at a test frequency in the range of 9 to 10 Gc. The test method for dielectric constant proposed by the ASTM specifies a sample diameter of 0.04 inch for this frequency range. As noted above, the samples reported in this paper had diameters of approximately 0.035 in. This value was chosen before the ASTM diameter was established.

The equivalent circuit of a dielectric post in a rectangular waveguide is given by Marcuvitz.[4] For a sample with a diameter of 0.035 in at a frequency of 9.4 Gc, the sample behaves like a shunt capacitive reactance whose normalized magnitude is given in

$$X' = \frac{67.1}{K' - 1} - 0.47. \tag{6}$$

[4] N. Marcuvitz, "Waveguide Handbook," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 10, p. 266; 1951.

If the equation for shunt reactance had been derived on the basis of perturbation theory, the result would be as shown in (7)

$$X' = \frac{67.1}{K' - 1} \cdot \tag{7}$$

Eqs. (6) and (7) are both approximate. However, (6) is the more accurate since it contains the term calculated on the basis of perturbation theory, and also includes an additional term. With this additional term, the normalized reactance goes through zero and becomes negative for sufficiently large but finite values of K'. However, in the case of (7), the normalized reactance cannot become negative no matter how large K' is. Reference to the tabulation of normalized reactance given by Marcuvitz[5] shows that the normalized reactance does indeed go through zero and becomes negative as K' increases.

It can be shown that the error in calculating K' using (3) is substantially the same as in calculating X' using (7). A first-order correction to the perturbation theory can therefore be obtained from (6) and (7). Manipulation of these equations yields

$$\frac{(K'-1)_T}{(K'-1)_A} = \frac{1}{1 + 0.00705(K'-1)_A} \cdot \tag{8}$$

$(K'-1)_T$ is the "true" value of $(K'-1)$. $(K'-1)_A$ is the "apparent" value of $(K'-1)$ that would be obtained through use of (3).

Eq. (8) applies only for a diameter of 0.035 inch. A correction graph to cover the range of diameters actually encountered is shown in Fig. 1. The data given in Table I incorporates this correction.

Effect of Finite Resistivity of Cavity Walls and Cavity Irises on Measurement of Tan δ

In using perturbation theory in the derivation of (4), the only source of losses that was considered was the loss introduced into the cavity by the sample. Losses present in the cavity itself and the variation of these losses with frequency were not considered. We will now determine the effect of cavity losses on the measurement of tan δ. The cavity will always be assumed to be at resonance.

In the following discussion, we will assume that when a sample is inserted into the cavity it has no dielectric losses. In this case, the only losses in the cavity will be due to the finite resistivity of the cavity walls and the losses coupled into the cavity through the two cavity irises. We can then write

$$\tan \delta_c = \tan \delta_\mu + \tan \delta_r, \tag{9}$$

[5] *Ibid.*, see p. 267.

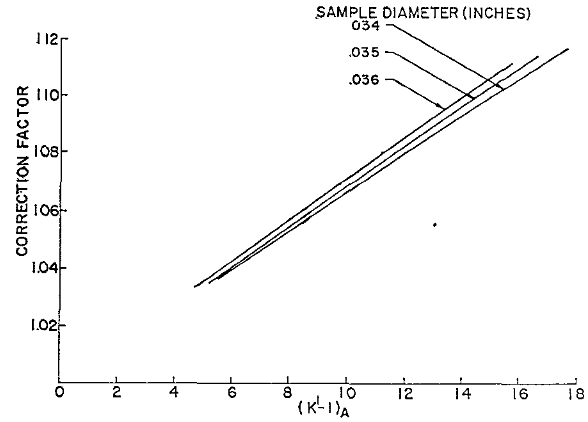

Fig. 1—Correction factor for calculation of $(K-1)$. Divide the value of $(K'-1)$ obtained from (3) by the correction factor to get the corrected value.

where tan δ_μ is the contribution of the finite resistivity of the cavity walls to tan δ_c, and tan δ_r is the contribution due to the two irises. A criterion as to whether the cavity losses cause an error in the measurement of tan δ is given in (4). For a lossless sample, we should have $(\tan \delta_c)_2 - (\tan \delta_c)_1 = 0$. Thus, if $(\tan \delta_\mu + \tan \delta_r)_2 - (\tan \delta_\mu + \tan \delta_r)_1 = 0$, the cavity losses do not cause an error in the measurement of tan δ. However, if the difference between the above two terms is not zero, then the cavity losses will cause an error in the measurement of tan δ.

A term that will be used in the following discussion is T, the transmission loss of the cavity. The transmission loss is defined as the ratio of the waveguide output when the cavity is omitted, to the output when the cavity is present. It is readily shown that when the two irises have the same dimensions (as was the case in the cavity used in these tests), we have

$$T = \frac{\tan \delta_c}{\tan \delta_r} \cdot \tag{10}$$

Finite Resistivity of Cavity Walls

$(\tan \delta_\mu)_1$ can be determined by calculating the energy stored in the magnetic field in the cavity, and the energy lost per sec due to the finite resistivity of the cavity walls. Applying the definition of the loss tangent of a circuit element,[6]

$$\text{loss tangent} = \frac{\text{energy lost per sec}}{2\pi F \cdot \text{energy stored in circuit}}, \tag{11}$$

we obtain

$$(\tan \delta_\mu)_1 = A \frac{F^2 + B}{F^{5/2}} \cdot \tag{12}$$

[6] E. C. Jordon, "Electromagnetic Waves and Radiating Systems," Prentice-Hall, Inc., Englewood Cliffs, N. J., p. 291; 1950.

A and B are functions of the waveguide dimensions only.

When a lossless sample is placed in the cavity, the resonant frequency is reduced, and the losses due to the finite resistivity of the cavity walls are represented as $(\tan\delta_\mu)_2$. Let us define $\Delta(\tan\delta_\mu)$ as $(\tan\delta_\mu)_2-(\tan\delta_\mu)_1$. To a good approximation $\Delta(\tan\delta_\mu)$ can be obtained by taking the differential of $(\tan\delta_\mu)_1$, as given in (12), with respect to frequency. We then have,

$$\Delta(\tan\delta_\mu) = -\frac{\Delta F}{2F}(\tan\delta_\mu)_1\frac{1+5B/F^2}{1+B/F^2}. \quad (13)$$

Using (3), (4), (9), and (10) and making the approximation that $K'\approx K'-1$, it is readily shown that the error in $\tan\delta$ due to the finite resistivity of the cavity walls is given by

$$(\text{Error in }\tan\delta)_\mu = -\frac{(\tan\delta_c)_1}{4}\cdot\frac{T-1}{T}\cdot\frac{1+5B/F^2}{1+B/F^2}. \quad (14)$$

For the TE_{101} cavity used in these tests, $(\tan\delta_c)_1 = 3\times10^{-4}$, $T=3$, $F=9.4$ Gc, and the magnitude of B/F^2 was less than 0.01. Thus the effect of the finite resistivity of the cavity walls was to make the measured values of $\tan\delta$ too low by approximately 0.5×10^{-4}.

For a TE_{10n} cavity, and $F=9.4$ Gc, $B/F^2=0.22$ for $n=3$, and 0.43 for n very large.

Cavity Irises

We will first determine the resistance R_s coupled into the cavity by one of the irises. The iris was a small hole centered in a thin transverse metallic plate as shown in Fig. 2(a). It is assumed that the input and output irises have the same dimensions.

The equivalent circuit of a transmission line containing a cavity is shown in Fig. 2(b). R_u is the resistance due to the finite resistivity of the cavity walls. L and C are equivalent inductance and capacitance of the cavity. X is the equivalent reactance due to an iris. According to Marcuvitz[7]

$$\frac{X}{R_0} = C_1[F^2-F_c^2]^{1/2}. \quad (15)$$

R_0 is the waveguide impedance, F_c is the cutoff frequency of the waveguide, and C_1 is a constant that is independent of frequency.

The equivalent circuit of the cavity at resonance is shown in Fig. 2(c). R_s is the resistance coupled into the cavity due to one iris. Using Thevenin's theorem and (15), it can readily be shown that

$$R_s = C_2R_0[F^2-F_c^2] = C_3F(F^2-F_c^2)^{1/2}, \quad (16)$$

[7] Marcuwitz, *op. cit.*, see p. 238.

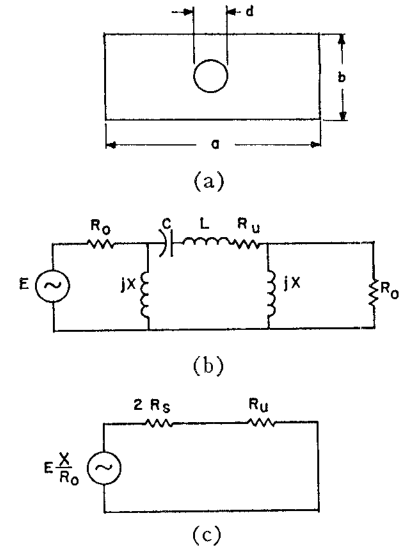

Fig. 2—(a) Iris plate. (b) Equivalent circuit of waveguide containing cavity. (c) Equivalent circuit of cavity at resonance.

where C_2 and C_3 are constants that are independent of frequency.

The energy lost per sec due to R_s is proportional to the square of the magnetic field at the iris. Using (11), it can be shown that

$$(\tan\delta_r)_1 = C_4\frac{(F^2-F_c^2)^{3/2}}{F^2}. \quad (17)$$

$(\tan\delta_r)_1$ is the loss tangent of the cavity without a sample in it, when only loss due to the two irises is considered, and C_4 is a constant independent of frequency. When a lossless sample is placed in the cavity, the resonant frequency is reduced, and the loss due to the cavity irises is represented as $(\tan\delta_r)_2$. Let us define $\Delta(\tan\delta_r)$ as $(\tan\delta_r)_2-(\tan\delta_r)_1$. To a good approximation, $\Delta(\tan\delta_r)$ can be obtained by taking the differential of $(\tan\delta_r)_1$ with respect to frequency. We then have

$$\Delta(\tan\delta_r) = \frac{\Delta F}{F}(\tan\delta_r)_1\frac{(F/F_c)^2+2}{(F/F_c)^2-1}. \quad (18)$$

Using (3), (4), (9), and (10), and making the approximation $K'\approx K'-1$, it is readily shown that the error in $\tan\delta$ due to the irises is given by

$$(\text{Error in }\tan\delta)_r = \frac{(\tan\delta_c)_1}{2T}\cdot\frac{(F/F_c)^2+2}{(F/F_c)^2-1}. \quad (19)$$

For $(\tan\delta_c)_1=3\times10^{-4}$, $T=3$, $F=9.4$ Gc, and $F_c=6.6$ Gc, the effect of the cavity irises is to make the measured value of $\tan\delta$ too low by approximately 2×10^{-4}.

Accuracy

Real Part of Dielectric Constant

Measurements of the rod diameter were made with a bench micrometer. All samples had some taper along the length and some ellipticity about the cross section. It is estimated that the effective diameter was measured with an accuracy of $\frac{1}{2}$ per cent. This would contribute an error of 1 per cent in the measurement of K'.

The resonant frequency was determined by taking the average of the two frequencies at which the output was a given fraction below the resonant output. Since all frequency measurements were made using a frequency counter, this method is very precise. A maximum error of 2 per cent was attributed to the electrical measurement on the basis of spread in results using different cavities. The maximum over-all error in measuring K' was thus 3 per cent.

Tan δ

As noted above, the loss tangent of the cavity was determined by use of (5). The measurement was made twice using a different value of α each time. If the two different determinations disagreed by more than 3 per cent additional measurements were made. A frequency counter was used in making the measurements. The maximum over-all error in tan δ was taken as 0.0005.

General Synthesis of Asymmetric Multi-Element Coupled-Transmission-Line Directional Couplers*

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Summary—**An exact synthesis procedure is derived for a class of asymmetric multi-element coupled-transmission-line directional couplers with any number of elements. It is based on the equivalence between the theory of the directional coupler and that of a stepped quarter-wavelength filter. This can be treated using Richards' theorem for the synthesis of transmission-line distributed networks, as described previously by Riblet. The method is extended to give a general expression for the input reflection coefficient of the stepped filter, which corresponds to the voltage coupling of the directional coupler. Explicit formulas for the parameters of two, three, four and five couplers are derived and the extension to larger number of elements is straightforward. Two and three element couplers have been designed on this theoretical basis, and show excellent agreement with theory, for example a three element coupler of 20 db ±0.5 db over a 6:1 bandwidth, and a two element coupler of 3.2 db ±0.85 db over a 6.7:1 bandwidth. It is possible to design a 3-db ±0.43 db-coupler for decade bandwidths using only four elements. The 3 db-couplers may be used as 90° hybrids by careful choice of reference planes in the output parts.**

I. Introduction

COUPLED-TRANSMISSION-LINE directional couplers have been described by a number of authors (Oliver,[1] Jones and Bolljahn,[2] and Shimizu and Jones[3]). The simple quarter-wavelength directional coupler has perfect isolation and perfect input match, but the coupling varies according to

$$L = 1 + \tfrac{1}{4}(Z_{oe} - 1/Z_{oe})^2 \sin^2 \theta \tag{1}$$

where L is the power insertion loss from the input to arm 4 (Fig. 1). Z_{oe} and Z_{oo} are the impedances of the even and odd modes in the coupled line normalized to the impedance of the input lines, and are related by

$$Z_{oe}Z_{oo} = 1. \tag{2}$$

Eq. (1) indicates that a useful bandwidth of rather more than one octave is obtained from the simple quarter-wavelength section. Shimizu and Jones[3] have described how much greater bandwidths may be obtained by cascading three coupled-line sections to form a three-quarter wavelength coupler. Considerable simplification of their equation (10) for the coupling is possible (see Appendix I), leading to the formula

$$L = 1 + \frac{1}{4}\left[\left\{2\left(Z_{oe}' - \frac{1}{Z_{oe}'}\right) + \left(Z_{oe} - \frac{1}{Z_{oe}}\right)\right\} \cdot \sin\theta\cos^2\theta - \left(\frac{Z_{oe}'^2}{Z_{oe}} - \frac{Z_{oe}}{Z_{oe}'^2}\right)\sin^3\theta\right]^2 \tag{3}$$

where, as in (1), L is the insertion loss from the input

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[1] B. M. Oliver, "Directional electromagnetic couplers," Proc. IRE, vol. 42, pp. 1688–1692; November, 1954.

[2] E. M. T. Jones and J. T. Bolljahn, "Coupled-strip transmission line filters and directional couplers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-4, pp. 75–81; April, 1956.

[3] J. K. Shimizu and E. M. T. Jones, "Coupled-transmission-line directional couplers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 403–410; October, 1958.